NSF GRANT #DMI - 0200439
NSF PROGRAM NAME: Manufacturing Enterprise Systems

Models and Algorithms for Integrated Multi-Stage Production/Distribution Systems: Third Party Logistics

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Abstract: Managers are increasingly realizing the value of integrating production and logistics management at the same time as third-party logistics contracting is becoming a more widely utilized practice across many industries. Motivated by this observation, we first review a variety of models that focus on integrated production/distribution systems, and then develop models to analyze the optimal operation of a production-distribution system with stochastic demand under a fixed commitment transportation contract. We characterize the optimal policy structures, present a computational study, and summarize the practical implications.

1. Introduction: In many manufacturing supply chains, effective management of both production and distribution operations is critical in order to reduce the operating costs while maintaining high level of customer service. To minimize costs and achieve service objectives, it is crucial to coordinate the planning and scheduling of these key functions, and this is particularly important if the manufacturing and logistics operations are dependent upon each other, in the sense that transportation and distribution takes place between manufacturing activities. Indeed, this has been the focus of much of our research over the past several years.

On the other hand, outsourcing non-core competencies has now become a widely accepted practice across many industries. For a variety of reasons, including increasing global competition and a focus on efficiently utilizing available resources, this trend is likely to continue unabated in the future. Clearly, operating an in-house trucking fleet is a difficult task, and for many firms not a core competency. Thus, the last ten or fifteen years have seen a dramatic growth in the use of external logistics providers to meet the logistics needs of manufacturing firms. For example, a major medical imaging device maker that we have worked with outsources the movement of main components between its component manufacturing operation in Israel, and its assembly plant in Milwaukee. Similarly, Samsung currently produces large LCD panels at Tang-Jeong (70 miles south of Seoul), but produces the notebook computers that use those panels in Suwon (15 miles south of Seoul) and other facilities worldwide.

By outsourcing logistics and distribution, manufacturing firms can utilize economies of scale that a third-party logistics provider already has established and avoid the investment to build necessary infrastructure. On the other hand, the use of external logistics providers makes the coordination of production and logistics much more difficult as the firm now must account for shipping schedules and resource availability limitations that may be beyond its control.

In our ongoing research, we are investigating the structuring of third partly logistics transportation contracts, and the use of contracts to coordinate production and distribution operations within a supply chain. In particular, we are focusing on a class of contracts that specify frequency and volume of transportation capacity that will (or may) be required by a particular firm in advance. We explore a variety of questions relating to the offering, acquisition, pricing, and use of these contracts, both by suppliers and by customers. The overall goal of our research is to address a variety of important issues:

- **From the perspective of the buyer of logistics**

service, have explored or will explore a variety of issues:

  - What products and services should be acquired?
  - How much are these products and services worth?
  - How should these products be used in the context of manufacturing operations?

- **From the perspective of the seller of logistics service**, we will analyze the following:

  - What products should be offered?
  - What types of contracts should be committed to?
  - How should these contracts be priced?
  - What is the value of reducing demand variability through forward commitment or capacity reservation schedules?
  - What is the value of flexibility in providing logistics services?
  - What decisions are involved in selecting long term clients to work with?

In our initial research (reported in this paper), we have focused on the manufacturing firm who purchases logistics services. We have started to analyze the impact of outsourcing logistics services on the efficiency of production, and to develop an understanding of the costs and benefits of this outsourcing, in order to quantify the value of a variety of logistics contracts and services to the buyer. We are particularly interested in understanding the trade-offs between economies of scale in logistics, and the level of inventories that must be employed to take advantage of these economies of scale while minimizing overall operating cost. This is a particularly important issue for firms that have been moving toward just-in-time production environment through low inventory policies.

Once we complete our understanding of the manufacturing firm's behavior, we plan to study the value of logistics contracts to shipping firms (logistics providers) and develop models to help logistic providers determine the types of products she should offer and how to price each logistics service product. In particular, we are interested in developing a model in which a single logistics firm chooses a portfolio of products to serve her clients with finite capacity. Initially, we plan to focus on a single firm serving multiple manufacturing firms, but ultimately, we plan to extend our models to embrace multiple logistics providers competing in a marketplace.

2. Literature Review and Preliminary Results in Integrated Production and Distribution Models: This research builds on two streams of research that have developed over the last several years. The first involves integrated manufacturing and distribution models, and the second involves the development of supply contracts, typically focusing on production capacity or pricing issues. Although both issues have been studied in isolation, there has been very little exploration of models that consider supply contracts for distribution in a setting that integrates production and distribution – work which would integrate aspects of the two streams of research.

2.1. Integrated Production/Distribution Models: Although there is a vast literature on inventory management, transportation and supply chain management, there is a relatively small amount of work in which the coordination between production and distribution policy in a single model is considered, and very little work which considers finite production capacity and nonlinear distribution costs, although these are important features of many real manufacturing environments. We have previously employed a variety of approaches to develop insight into the impact of capacity and the cost of transportation for a broad range of multi-stage supply chains.

2.1.1. Deterministic Models: We initially considered a deterministic model of a two stage supply chain facing a deterministic stream of external demands for a single product. We assumed an infinite supply of raw materials at stage one, and capacitated production at both stages. Items are manufactured at stage one, and then held in inventory at stage one prior to shipping. Items are transported to stage two, where they are again held in inventory. Additional capacitated production is completed at stage two (that is, value is added to each item, but no new items are created), items are held in finished goods inventory after this stage, and this inventory is used to meet final demand. Each period, production levels in stage one and stage two, as well as transportation levels between stage one and stage two, must be determined.

Formally, in Kaminsky and Simchi-Levi [41] we consider a two-stage, n period model of a supply chain, illustrated in Figure 2.1.1. As described above, an infinite supply of raw material is available at stage one. Each period, $x_t^1, t = 1, 2, ..., n$ units are produced at stage 1, at a cost of c_i^t, where $x_t^1 \leq C_t^1$. Units can be held in inventory at the stage 1 buffer, where a holding cost $h_t^1, t = 1, 2, ..., n$ is charged per unit at period t. In addition, s_t units are also shipped to a buffer located before stage two (with no shipping lead time). If a shipment occurs, a fixed cost of f_t is charged, and a variable cost of v_t per unit is charged. Units can be held in inventory in the buffer before stage two, where a holding cost of h_t^2 is charged per unit. Alternatively, $x_t^2 \leq C_t^2$ units can enter production, at a cost of c_t^2 per unit. After production, items can be held in finished goods inventory at the post stage two buffer where a holding cost h_t^f is charged per unit, or they can be shipped to meet demand d_t. Also, we let d_{tn} represents the cumulative demand from t to period n, y_t be the shipment indicator variable, and i_t^1,i_t^2, and i_t^f be the inventory levels at the stage one buffer, at the pre stage two buffer, and at the post stage two buffer at the end of period t, respectively. We note that in this model, all demand must be met, and call this the two stage production distribution problem (2SPDP).

We model our problem as follows:

$$(P) \min \sum_{t=1}^{n} (i_t^1 h_t^1 + i_t^2 h_t^2 + i_t^f h_t^f + y_t f_t + s_t v_t + c_t^1 x_t^1 + c_t^2 x_t^2)$$

$s.t.$

$$x_t^1 \leq C_t^1, \quad t = 1, 2, \ldots, n \tag{1}$$
$$i_t^1 = i_{t-1}^1 - s_t + x_t^1, \quad t = 1, 2, \ldots, n \tag{2}$$
$$s_t \leq y_t d_{tn} \quad t = 1, 2, \ldots, n \tag{3}$$
$$x_t^2 \leq C_t^2 \quad t = 1, 2, \ldots, n \tag{4}$$
$$i_t^2 = i_{t-1}^2 - x_t^2 + s_t \quad t = 1, 2, \ldots, n \tag{5}$$
$$i_t^f = i_{t-1}^f + x_t^2 - d_t \quad t = 1, 2, \ldots, n \tag{6}$$
$$i_t^f, i_t^1, i_t^2, x_t^1, x_t^2, s_t \geq 0 \quad t = 1, 2, \ldots, n \tag{7}$$
$$y_t \in \{0, 1\} \quad t = 1, 2, \ldots, n \tag{8}$$

Constraints (1) and (4) are capacity constraints for both stages. Constraints (2), (5), and (6) are the inventory balance equations for the three buffers. Constraint (3) ensures that the fixed cost is incurred when items are shipped. We note that we have also **considered a general version** of this model, in which rather than being characterized by a fixed and a linear cost, the shipping costs can be a general concave cost function: $f_t(s_t)$.

In Kaminsky and Simchi-Levi [41], we developed efficient algorithms for the model described above, both with the fixed and linear transportation cost, and more general concave transportation costs. We characterized a variety of properties of the model that reduced the complexity of the problem, and developed a dynamic programming approach to solving this problem. By carefully avoiding duplication of effort, we developed an algorithm which allows us to solve the original problem in $O(n^4)$ time for fixed plus linear costs, and $O(n^8)$ algorithm for the constant capacity case with general concave production costs. Recently, van Hoesel et. al. [38] extended some of these results to more supply chain levels.

In addition to the work described above, there has of course been a long history of research into deterministic *single stage* single item lot sizing models, starting with the seminal works of Wagner and Whiten [61] for the uncapacitated model, and Florian and Klein [29] for the capacitated model. Aggarwal and Park [1], Federgruen and Tzur [24], and Wagelmans et al .[60] developed faster exact algorithms for the uncapacitated case, while Love [51] and Baker et al. [11] developed more general algorithms for the capacitated case. Various authors have considered multi-stage production models under deterministic *constant* demand. Muckstadt and Roundy [54] survey and summarize many of these results. The work presented in this paper allows for time-varying multi-period demand. A variety of research has been devoted to heuristic and mathematical programming-based methods for deterministic multilevel, multi-product lot sizing problems. Baker [10] summarizes various heuristic approaches which appeared in the literature up to that point, and subsequently, Harrison and Lewis [36], Katok et al. [49], and Armentano et al. [9] have proposed additional heuristics for these complex problems. Although these papers don't explicitly mention transportation costs, some of the formulations are general enough to encompass our model. In addition, Chan et al. [15] and the references therein consider a concave cost network flow version of a multistage inventory/transportation problem (without capacitated production), and develop heuristics for the problem. However, in contrast to the heuristics these authors have focused on, we focused on polynomial approaches to obtain the optimal solution.

Finally, Zangwill [63], and Crowton and Wagner [20] develop optimal dynamic programming-based algorithms

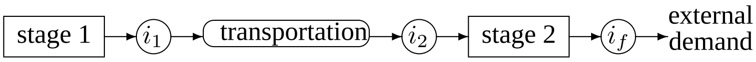

Figure 1: Model 2SPDP

for uncapacitated multi-stage lot sizing problems without considering transportation. Related approaches are also described in Zipkin [66].

2.2. Stochastic Models: While the results from our deterministic model characterizes some of the qualitative structure of the optimal policy, which will help us to develop efficient heuristics, the analysis of stochastic versions of these models provides valuable insight into impact of uncertainties in the system such as random demands and finite random capacities. To capture essential features of the canonical examples which motivated this stream of research, we consider two different settings: continuous time stochastic models and discrete time stochastic models. Discrete time models capture those situations in which system information can only be reviewed or acted on at certain fixed times (daily, weekly, etc), whereas continuous time models are better suited to systems with advanced information technology that allows for continuous monitoring of system variables, with action at an appropriate time. In addition, both classes of models are amenable to different analysis techniques, and thus by exploring both classes of models, we were able to investigate different system characteristics.

There has, of course, been a variety of literature on multi-stage inventory systems. Clark and Scarf [19] show that a base stock policy is optimal in a finite horizon periodic problem with no fixed cost and no capacity constraint. Federgruen and Zipkin [25, 26] show that the optimality of the order-up-to policy is still valid in the case of discounted and average infinite horizon problems. Chen [16, 17] demonstrates the effectiveness of the (R, nQ) policy in a similar setting, while Chen and Song [18] characterize the structure of the optimal policy in a multi-stage inventory model with Markov modulated demand. A few authors have considered the control of capacitated production systems. Federgruen and Zipkin [27, 28] consider an inventory policy in a capacitated production system under stationary demand with no fixed cost. Glasserman and Tayur [32, 33, 34] study a multistage stationary production-inventory model with capacitated production and demonstrate the effectiveness of IPA (infinitesimal perturbation analysis) as a tool to obtain an optimal policy. Kapuscinski and Tayur [48] extend the model to periodic demand. Parker and Kapuscinski [55] consider a periodic review problem of a two-stage capacitated echelon inventory system with zero or deterministic leadtime and show that the optimal policy is a modified base stock policy. However, in their model, the replenishment decision is free of economies of scale in shipping quantity so that it is reasonable to conjecture that a variant of an order-up-to policy should be optimal. In most of these papers, the optimal policy is either a base stock policy, or a policy with monotone structure. An exception to this is Frank et al. [31], who consider a periodic review model with fixed cost and lost sales when the system has "must-meet" deterministic demand and random demand at each period. They show that the optimal shipping quantity and rationing policy are not necessarily monotone, because the inventory on hand may not satisfy existing demands, and can be conserved for deterministic demand in subsequent periods.

There has been another stream of literature on the control of production systems. Ha [35] characterizes the structure of the optimal policy in make-to-stock system as a switching curve type policy. Carr and Duenyas [14] consider a make-to-stock/make-to-order system. Ahn et al. [2], Iravani et al. [39], Duenyas and Patane-Anake [21] characterize the optimal policy in a serial production line with single or multiple resources. Duenyas and Tsai [22] consider a two-stage production system where stage one can sell its finished goods and provide the components to the downstream stage at the same time. However, in contrast to our research, very little research considers multiple units shipped at a fixed cost.

2.2.1. Continuous Time Stochastic Models: We first consider a two stage supply chain with the random demands and the finite capacity at each stage as in Figure 2.2.1. The orders arrive one at a time at stage two according to a Poisson process with rate λ. Two separate operations, which take place at different locations, are required to covert the raw materials to finished goods. As with our deterministic model, our general stochastic model assumes capacitated production at each stage. A single server whose processing time follows i.i.d exponential distribution with rate μ_i, $i = 1, 2$ is available at each of the two stages. Items are produced at stage one, the push stage, and can then be either held as inventory at stage one, or shipped to stage two, in which case fixed shipping cost are incurred. Items will be processed at stage two, the pull stage, to satisfy the outstanding orders. Holding costs are incurred on any intermediate inventory, and penalty costs are incurred for orders waiting to be filled. The holding cost is incurred at a rate h_i ($i = 1, 2$) per unit time per item, while the backorder cost is incurred at rate b per unit time per outstanding order until the demand is met. To capture the added value after each operation, we assume that these costs are non-decreasing (i.e., $h_1 \leq h_2 \leq b$). Shipping is assumed to be immediate (although we believe that the qualitative result will not change even in the cases of non-zero transportation time), and a fixed cost, K, is incurred for each shipment to reflect economies of scale. Also, let n_1, n_2, and n_3 represent the inventory at stages one and two, and the outstanding orders, respectively. We model this problem as a **Markov Decision Process** with the objective of minimizing the long run average cost, and characterize the optimal policy that coordinates the production and distribution decisions.

Our results show that the trade off between the saving from economies of scale in shipping, and additional holding costs due to shipping, leads to a complex optimal policy. As Figure 3 and 4 illustrate, the structure of an optimal policy can be counter-intuitive to what most previous work suggests.

As Figure 3 indicates, the optimal shipping quantity is not always monotone in the amount of on-hand inventory at stage one or the number of outstanding orders. In this example, for instance, it is optimal to ship 22 units at state (24,0,3), but 20 units at state (35,0,3), even though this second state has more units to ship from stage one. Furthermore, the optimal shipping quantity is not monotone in the number of outstanding orders. In fact, the optimal shipping quantity cycles with increasing n_3. This result is surprising, since it implies that in some cases it is optimal to ship a smaller quantity as the number of outstanding orders increases. Similar counter intuitive behaviors exist in the optimal production policy. In Ahn and Kaminsky [6], we prove that for any state (n_1, n_2, n_3), it is optimal not to ship as long as $n_2 > 0$. In addition, for a version of this model in which shipping is capacity constrained, we show that for any state $(n_1, 0, n_3)$ such that $n_1 \geq Q$ and $n_3 \geq Q$, it is optimal to ship independent of production policy. However, extensive computational studies demonstrated that even in this restricted case, counterintuitive behavior made the behavior of this system difficult to characterize, with shipping and production quantities non-monotonic in most state variables and combinations of state variables.

Based on our analytical and numerical results, we have developed an effective heuristic for this model, called the (L, Q, α) policy. Under this policy, when inventory at stage 2 minus backorder level falls below L, the firm produces to raise the inventory to Q. Also, if $n_2 = 0$ and the inventory at stage 1 plus α times the backorder level is greater than or equal to Q, the firm ships the minimum of Q and the available inventory at stage one to stage two. In Ahn and Kaminsky [6], we discuss how to choose these critical numbers in detail, we computationally test the effectiveness of this heuristic, and show that in many cases, it is within a few percentage points of optimal.

2.2.2. Discrete Time Stochastic Models: We also consider a two-stage manufacturing system where a fixed setup cost (in addition to a proportional cost) is charged for transportation of items and both production and transportation decisions are made after periodic review. There is no fixed cost for production in either stage, however there is production capacity at the second stage. Items are manufactured at stage one, and then held in inventory at stage one prior to shipping. Transported items are held in inventory at stage two until additional production is completed and then the external demand is met.

As in the continuous time model, the policy problem is to determine the production level at first stage, as well as the transportation level between stage one and stage two at each period. Notice that the first stage is operated in a push-based manner (i.e., make-to-stock) utilizing the economies of scale in transportation. On the other hand the second stage, which keeps inventory for only interim products, is clearly a pull-based stage (i.e., make-to-order) since the production level is directly determined by the external demand. Although production is unrestricted at stage one, shipment from stage one to stage two is restricted by the amount of inventory at stage one. Similarly the production at stage two is limited by the amount of inventory at that stage plus the shipment from stage one as well as the production capacity at stage two. The planning horizon is finite. The demand is stochastic and independent from period to period. In each period, production and shipping decisions are made before demand is realized.

As in the continuous time model, we formulate this system as a discrete time Markov decision process. In what follows, $i \in \{1, 2\}$ represents the production stage, and $t \in \{1, ..., T\}$ represents the time period.

I_t^i: On-hand inventory at stage i at the beginning of period t.

B_t: Backlog at stage two at the beginning of period t.

x_t^i: Production level at stage i at period t.

s_t: Shipment level from stage one to stage two at period t.

d_t: External demand (random disturbance) realized at period t.

c_i: Unit production cost at stage i.

h_i: Unit holding cost of on-hand inventory at stage i.

p: Unit penalty cost of backlogged demand.

C: Production capacity at stage two.

K: Fixed cost charged for shipping.

α: Discount factor.

Let d_t be demand of end product in period t. Note that since there is a finite production capacity at stage two, it is possible to have positive on-hand stage two inventory and positive backlog at the same time. Hence we represent the system state by $\vec{I_t} = (I_t^1, I_t^2, B_t)$, and resultant state space by $S_t \subseteq Z_+^3$, $\vec{I_t} \in S_t$. The decisions that need to be taken at each period are the production at stage one and shipment. The production at stage two is determined by the demand, backlog, inventory, shipment, and the production capacity, and follows the following rule; $x_t^2 = min(B_t + d_t, I_t^2 + s_t, C)$. Let $\vec{u_t} = (x_t^1, s_t) \in C_t$ be an action in period t and $C_t \subseteq Z_+^2$ be the associated action space. Here, the set of admissible actions is actually state dependent. For example, the shipment at each period is limited by the on-hand inventory at stage one. Let $U_t(\vec{I_t}) = \{(x_t^1, s_t) \in C_t | s_t \leq I_t^1\}$ be a set of admissible controls in state $\vec{I_t}$. A policy (control law) in this problem is a sequence of control functions, $\pi = \{\mu_0, ..., \mu_{N-1}\}$. For every state in the state space, a control function determines the optimal control, $\vec{u_t} = \mu_t(\vec{I_t})$, such that $\mu_t(\vec{I_t}) \in U_t(\vec{I_t})$ for all $\vec{I_t} \in S_t$. The system evolves as a function of the current state, control, and the random disturbance. The system function is defined by $\vec{I}_{t+1} = f_t(\vec{I_t}, \mu_t(\vec{I_t}), d_t)$, where

$$f_t((I_t^1, I_t^2, B_t), (x_t^1, s_t), d_t) = [I_t^1 + x_t^1 - s_t, I_t^2 + s_t - x_t^2, B_t + d_t - x_t^2].$$

The (expected) cost in each period is determined by the state, control, and random demand. Let $g_t(\vec{I_t}, \vec{u_t}, d_t)$ be the expected one-period cost function,

$$g_t((I_t^1, I_t^2, B_t), (x_t^1, s_t), d_t) = K\mathbf{1}_{(s_t > 0)} + x_t^1 c_1 + x_t^2 c_2 + \\ (I_t^1 + x_t^1 - s_t)h_1 + E[(I_t^2 + s_t - x_t^2)h_2 + (B_t + d_t - x_t^2)p].$$

Given an initial state, $\vec{I_0}$, the policy problem is to minimize the total system expected costs,

$$\min_\pi J_\pi(\vec{I_0}) = \sum_{t=0}^{N-1} \alpha^t g_t(\vec{I_t}, \mu_t(\vec{I_t}), d_t).$$

If we let J_t be the minimum expected cost-to-go function at period t, then the Bellman equations for this system are:

$$J_t(\vec{I_t}) = \min_{\vec{u_t}} \{g_t(\vec{I_t}, \vec{u_t}, d_t) + \alpha E[J_{t+1}(f_t(\vec{I_t}, \vec{u_t}, d_t))]\} \\ \text{for } t = 1, .., N-1 \text{ with } J_N(\vec{I_N}) = 0$$

To gain insight into the structure of optimal policy for this model, we performed computational testing using the value iteration algorithm with a variety of parameter values. For the value iteration, we truncated the state space with a conservative upper bound, redirecting any transition to a state outside the truncated space to the nearest state in. Through an extensive computational study, we observed that neither optimal production nor shipping decisions are monotone in most of the state variables. In particular, by exploring various examples, we observed that the production level at stage one **may not be monotone** in outstanding orders and stage-1 inventory level, and that the optimal shipment is **not monotone** in outstanding orders.

Clearly, the optimal policy is not only complex but also **counterintuitive**. This counterintuitive behavior seems to stem from the nonlinearity of the shipping cost. The production capacity at stage two further complicates the structure of the optimal policy. The lack of monotonicity in both the shipping and production levels makes the structural properties of this model very difficult to determine. Thus, we are motivated to consider various modifications and simplifications of this model analytically and computationally. In particular, we will focus on the performance of easily implementable monotone heuristics, which includes

i) a policy which restricts shipping to either zero or $\min[\text{Inventory}, F]$ for $F > 0$.

ii) a policy which restricts production or shipment or both to an (s,S) type policy.

iii) a policy which restricts shipping to either zero or some fixed quantity.

3. Third Party Logistics Contracting Models and Results: As we discussed in the introduction to this paper, outsourcing non-core competencies is becoming more and more prevalent across many industries, and in general, operating an in-house trucking fleet is a difficult task and not a core competency. In recent years, the number of firms that prefer to cooperate with outside companies for their logistics related responsibilities is growing rapidly. This cooperation is called third-party logistics in general and is explained in detail in Simchi-Levi et al. [57]. Hence, more and more firms are relying on contractual relationships with third party logistics providers. Thus, effectively designing these contracts, and coordinating production and distribution operations when constrained by these contracts, is of growing importance. Several types of supply contracts are most common in both practice and most often featured in the literature:

1. Long-term (or forward buy or fixed commitment) contracts: Both buyer and seller specify the volume and schedule of future shipments in advance.
2. Option contracts: The buyer pays a fee (premium) to reserve service capacity within a certain time frame and then pays an execution (or exercise) price when products are being shipped.
3. Flexible contracts: A fixed amount of supply is determined when the contract is signed, but the specific time and amount to be delivered in each shipment can change within a pre-negotiated limit. These contracts are used in practice to share risk between the parties, and are equivalent to a relevant combination of a long-term contract and an option contract.

Contract design problems have been extensively studied in the economics and operations management literature. Rothkopf and Harstad [56] present a review and general description of competitive bidding models in economics literature. The studies in this area deal with aspects such as determination and exploitation of the contract types and their implication on the parties involved in the contract, motivation of contractual structures, the legal issues in contracting environments, and selection of the contractor. The differences of the supply chain contracting literature from the economics literature is that the former focuses on some operational details such as the material flow among the parties, uncertainties in demand or supply, and penalties charged. Tsay et al. [59] provide a survey of recent developments in supply chain contracts.

Henig et al. [37], Yano and Gerchak [62], and Ernst and Pyke [23], among others, investigate transportation contracts in supply chain environments, although in very different contexts from our own.

3.1. Model Development and Analysis: We have started to explore the effective operation of supply chains in the presence of logistics contracts. To do so, we have analyzed three stylized models with increasing complexity. Initially, we are interested in the operational issues regarding the shipment decisions under a transportation contract. In order to concentrate on shipment policies and for simplicity, we will initially ignore the production operations. However production with linear cost structure does not effect the analysis presented in this section. Our aim in this analysis is to gain useful insights that will hopefully help solving more complicated models. Below, we present our model and initial results.

Suppose a firm has infinite amount of supply available. It receives independent and identically distributed orders $w_t \geq 0$ at each period, $t = 0, ..., T-1$. At period 0, the firm is already in possession of a transportation contract for period T with unlimited capacity. There is a penalty cost p per unit of pending orders per period. There is also an option to ship orders with expedited shipping, which costs c per unit. The interest rate is $r > 0$, and the corresponding discount factor is $\alpha = (1+r)^{-1}$. The decision problem is to find the optimal level of expedited shipping u_t at each period given the number of pending orders x_t. The Bellman equations for this problem are:

$$J_T(x_T) = 0 \qquad \text{(P1)}$$
$$J_t(x_t) = \min_{u_t \in [0, x_t]} \{cu_t + p(x_t - u_t) + \alpha\, \mathbb{E}_{w_t}[J_{t+1}(x_t - u_t + w_t)]\}$$
$$t = 0, ..., T-1.$$

We have proved the following:

Proposition 1 *(Optimal Policy of (P1)) In* (P1), *the opti-*

mal shipping quantity function, μ_t, is given by:

$$\mu_t(x) = \begin{cases} x, & \text{if } p\frac{1-\alpha^{T-t}}{1-\alpha} > c; \\ 0, & \text{otherwise} \end{cases} \qquad t = 0, ..., T-1. \quad (1)$$

Next, we consider the case where the contract in (P1) is capacitated with $C \geq 0$, and the pending orders in excess of C at period T are shipped via expedited shipping. Then the Bellman equations are:

$$\begin{aligned} &J_T(x_T) = c(x_T - C)^+ && \text{(P2)} \\ &J_t(x_t) = \min_{u_t \in [0,x_t]} \{cu_t + p(x_t - u_t) + \alpha\,\mathbb{E}_{w_t}[J_{t+1}(x_t - u_t + w_t)]\} \\ &t = 0, ..., T-1 \end{aligned}$$

where we use the notation $(x)^+ \equiv \max(0, x)$. Notice that if $p \geq c$ then $\mu_t(x) = x$, or if $(1-\alpha)c \geq p$ then $\mu_t(x) = 0$, $t = 0, ..., T-1$. Thus, from now on we assume that $p < c$ and $(1-\alpha)c < p$ to avoid trivial cases. We also naturally assume that $x_0 \geq 0$. For this more advanced model, we show:

Proposition 2 *(Optimal Policy of (P2)) In* (P2) *for a given C, there is a sequence of increasing nonnegative numbers $\{R_t\}_{t=0}^{T-1}$ between 0 and C such that $\mu_t(x) = (x - R_t)^+$ for $t = 0, ..., T-1$.*

To understand how R_t changes with C, we will make the dependence of J_t, G_t, and R_t to C explicit by defining them as a function on C for the next proposition.

Proposition 3 *In* (P2)*, $R_t(C)$ is increasing in C, $t = 0, ..., T-1$.*

This result leads to the following:

Corollary 1 *In* (P2)*, $R_t = 0$ for all t such that $p\frac{1-\alpha^{T-t}}{1-\alpha} > c$ is satisfied.*

Finally, we consider the case where there are n time periods, T_j, in which the firm has contracts capacitated by C_j, $j = 1, ..., n$. The planning horizon is T_n and the Bellman equations are:

$$\begin{aligned} &J_{T_n}(x_{T_n}) = H_{T_n}(x_{T_n}) && \text{(P3)} \\ &J_t(x_t) = \min_{u \in [0,x_t]} \{H_t(u) + p(x_t - u) + \alpha\,\mathbb{E}[J_{t+1}(x_t - u + w_t)]\} \\ &t = 0, ..., T_n - 1 \end{aligned}$$

where $H_t(x) \equiv \begin{cases} c(x - C_t)^+, & \text{if } t \in \{T_1, ..., T_n\} \\ cx, & \text{otherwise.} \end{cases}$

We have been able to prove the following results for this model, and to characterize the optimal solution:

Lemma 1 *In* (P3) *value functions, $J_t(x)$, are increasing in x for all t.*

Proposition 4 *(Optimal Policy of (P3)) In* (P3) *for a given set of C_m and T_m, $m = 1, ..., n$, there is a sequence of sequence of increasing nonnegative numbers $\{\{R_t\}_{t=T_m+1}^{T_{m+1}}\}_{m=0}^{n-1}$ such that*

$$\mu_t(x) = \begin{cases} (x_t - R_t)^+, & \text{if } t \notin \{T_1, ..., T_n\} \\ \begin{cases} x, & \text{if } x \leq C_t \\ C_t + (x - R_t)^+, & \text{otherwise} \end{cases} \end{cases} \qquad (2)$$

where we define $T_0 \equiv -1$.

Proposition 5 *In* (P3)*, $R_t(\mathbf{C})$ is increasing in $\mathbf{C}$ for $t = 0, ..., T_n - 1$, where vector $\mathbf{C}$ is defined by $\mathbf{C} \equiv (C_1, ..., C_n)$.*

Corollary 2 *(Decomposition of (P3)) In* (P3)*, suppose that $p\frac{1-\alpha^{T(t)-t}}{1-\alpha} > c$ is satisfied for some t, where $T(t) = \min_{j=1,...,n}(T_j | T_j > t)$. Then*

$$R_t = \begin{cases} 0, & \text{if } t \notin \{T_1, ..., T_{n-1}\} \\ C_t, & \text{otherwise.} \end{cases}$$

These results allow us to develop effective techniques for numerically analyzing the performance of firms operating effectively under a variety of different system and contract parameters. We intend to complete this analysis as one of the initial stages of our research.

3.2. Computational Study: We have completed an extensive computational study, and in this section we summarize our key observations, which provide some insight about the behavior of the optimal policy with respect to changing problem parameters. Firstly, we note that the ratio p/c and its size relative to the discount factor α are the key factors determining the ship-down-to levels, R_t. This is somewhat expected since the asymptotic behavior of the optimal policy as the reserved capacity goes to infinity is solely determined by these parameters. The smaller this ratio or the discount factor, the larger and flatter the $\{R_t\}$ sequence, which consequently renders a transportation contract more attractive for the manufacturer. Secondly, we observe that the mean of the demand distribution normalized with the reserved capacity has a scaling effect. In other words as long as its ratio with the reserved capacity stays constant it does not effect the shape of the $\{R_t\}$ sequence. If this ratio increases, the sequence becomes steeper, and if it decreases, the sequence becomes flatter. In general, the more flat the ship-down-to levels, the more profitable it is to use a contract. Thirdly, the reserved capacity has a vertical shifting effect on the ship-down-to-levels. An increase in the capacity evenly increases the positive portion of the $\{R_t\}$ sequence, and a decrease does the exact opposite. Lastly, we observed that the ship-down-to levels increase with increasing dispersion of the demand distribution when p/c ratio is small, making the sequence more flat; when p/c ratio is large, it makes the positive part of the $\{R_t\}$ sequence more concave, increasing the ship-down-to levels for the early periods and decreasing for the later.

4. Conclusion and Future Research Directions: In this paper, we have completely characterized the optimal policy structure of a production-distribution system under a fixed commitment transportation contract when the production cost is linear and the production lead time is zero. Our analysis and numerical study show that, for a manufacturing firm that wants to outsource its logistics operations, it is significantly more attractive to buy a transportation contract rather than using just the spot market when the ratio of pending order penalty to expedited shipping cost is small. However, when this ratio is large, especially in the case of large demand variance, a substantial increase in the frequency of shipments is necessary for the contact to be useful, which would probably render the contract price too expensive to be profitable.

In the future, we intend to continue to explore the issues raised in the introduction to this paper. Initially, we plan to extend this study to the case where there are positive lead times, economies of scale and capacity for production, which will make the production-distribution integration problem more complex and realistic. We also plan to analyze the effect of some other contract types such as option and flexibility contracts in this setting. Next, we will include contract procurement problem in the model. Given any price structure on the contract parameters, it is possible to incorporate the contract procurement problem in the manufacturer's model using the results obtained in this study. Finally, we will explore the the problem from the contract providers point of view and model its resource allocation, contract design and pricing problems in a competitive market.

In conclusion, the models we analyzed in this study provide some insight to the practical use of transportation contracts and the foundation for future investigation of models that incorporate even more critical aspects of important real-world problems relating to integrated production and distribution management in the presence of third party logistics contracts.

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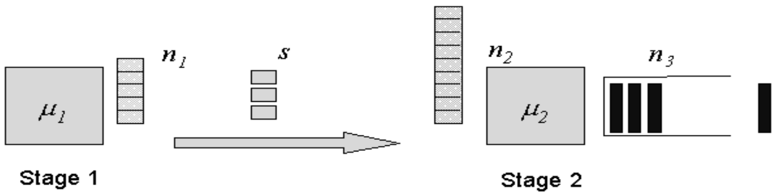

Figure 2: Stochastic 2-stage Push-Pull System

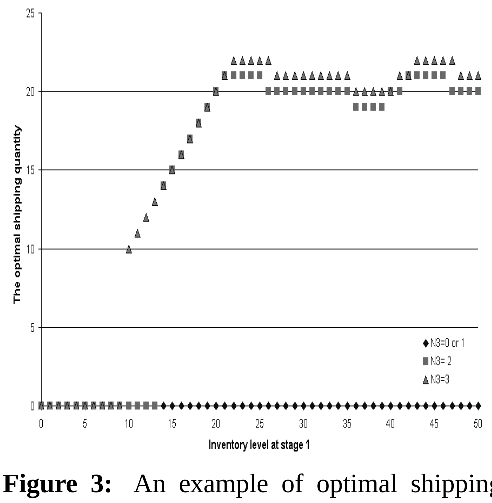

Figure 3: An example of optimal shipping quantity when $n_3 = 0, 1, 2$ and 3.

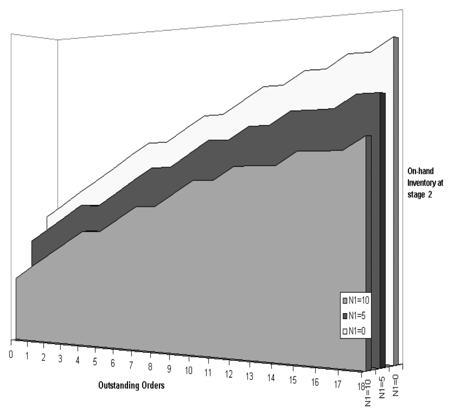

Figure 4: An example of optimal production policy at stage 1 when $n_1 = 0, 5$ and 10