A Multi-stage Decomposition Heuristic for the Container Stowage Problem

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A significant and growing volume of global trade travels on container ships, and each year larger ships are introduced. For this reason, the Container Stowage Problem, a fundamental problem in marine cargo transportation involving the optimal assignment of shipping containers of various types to specific storage locations in container ship at each port in order to maximize loading and unloading efficiency and minimize shipping costs, is growing in importance. In spite of this, the problem is currently solved in an ad-hoc manner based on experience and rules-of thumb. In this paper, we develop a multi-stage decomposition heuristic for this problem that accounts for the many complexities of real-world versions of the problem. This approach is performing well in testing on real-life problems, and is currently being developed for inclusion in commercial software.

1 Introduction

About 90% of the world's traded goods travel via the ocean, and about 90% of non-oil, non-bulk goods travel aboard container ships.[1] These ships carry their cargo in truck-sized containers, and with the growth of containerization, it is now possible to transport goods from a specific location to virtually any destination in the world without unpacking and repacking the container that is loaded at the origin. Indeed, container ships are credited with dramatically lowering the cost of ocean transportation, and containerization has reduced the cost of handling goods at ports, standardized and streamlined the logistics process, and enabled the cost-effective manufacturing of goods far from where they are consumed. The demand for ocean transportation of goods has grown dramatically with the rise of containerization, and ship builders and shipping lines are introducing more and larger ships to deal with this increase in demand. Indeed, the total container ship capacity has grown approximately 15 to 20 percent every year since 1968, and the capacity of the largest ship has grown approximately 5 percent a year since 1968.[2] These trends appear to be accelerating.

[1] http://uk.reuters.com/article/governmentFilingsNews/idUKL1018528520080110

[2] http://www.containership-info.net.tc

There are many challenging operational problems associated with container shipping, and these problems are only growing in importance and difficulty as ships grow in capacity. One of the most difficult of these is the Container Stowage Problem (CSP). Container ships typically visit a series of ports on a voyage, at each of which some containers are removed from the ship, and others are stored. Container ships have specific storage locations, called slots, in which containers are stored for transportation. Containers are stacked in these locations, so that containers on the bottom of the stack cannot be removed unless the containers stowed above them are already moved. In the CSP, each container to be stored on the ship needs to be assigned a specific storage location (known as a stowage plan) in such a way that a weighted combination of objectives is optimized subject to a set of constraints on a feasible stowage plan. Since actions at ports impact actions at future ports on a voyage, a multi-port stowage plan is critical to minimize total voyage costs and effectively utilize resources.

Although there are many possible objectives that could be considered, in discussions with our industrial partners, we elected to focus on three key objectives:

Cost of overstows As mentioned above, containers are stored in stacks on the ship, and thus if a container at the bottom needs to be removed, all of the containers on top of it also need to be removed. Containers that have to be removed and replaced at a particular port in order to reach containers below them are known as overstows. Port terminal authorities generally charge a fixed and known amount per rehandle or move of a container (although these charges can be different at different ports). Given a particular multi-port stowage plan, the cost associated with overstows can be calculated.

Fuel consumption Tiemroth [2006] identifies three classes of physical constraints (associated with weight distribution on the vessel), known as static constraints, that impact the final stowage plan. These include attitude constraints, stability constraints, and strength constraints, and all appear as constraints in our model. While it is possible that each of them to some extent impacts the fuel consumption of the ship, our industrial partners believe that the impact of container stowage on vessel trim (trim is the angle of a ship relative to the water level) has the most significant impact on fuel consumption. In particular, there is a predefined level of trim for each ship that minimizes fuel consumption, and every unit deviation from this level increases fuel consumption rate by a known percentage.

Crane utilization The effective utilization of port cranes directly impacts the time that the ship must spend in port, and thus the costs associated with keeping the ship in port. As ships get bigger, it takes longer to load and unload them, and thus it becomes even more critical to develop stowage plans that facilitate faster loading and unloading. This issue is also becoming increasingly important because most ports are already operating at or near full capacity and the expansion in port capacity does not appear to be matching the growth in total container ship capacity. Each port has more than one crane that can be dedicated to handling containers for a given container ship. The maximum throughput is achieved if all of the available cranes can work simultaneously and continuously (without interrupting each other) while loading and unloading containers. However, cranes may not be able to work in adjacent bays of a ship, cranes cannot pass each other, and it takes time for a crane to pass over the bridge of a container ship, so the relative location of the various containers that have to be removed at a given port can significantly impact crane utilization.

These objectives need to be considered subject to a variety of feasibility constraints (discussed in the next section of this paper). Traditionally, this problem is manually solved by planners, who attempt to optimize these objectives based on experience and rules-of-thumb. These planners are assisted by software that graphically represents the ship, calculates measurements associated with static constraints, and notifies them when potential plans do not meet static constraints. However, as ships grow in size, major shipping lines are realizing that this problem is becoming too big for planners to solve by hand. Our objective in this project is to develop a decision support tool to be embedded in the already existing graphical tool described above, to help planners with this difficult problem.

2 The Model and Literature Review

The CSP can be formally summarized as the problem of assigning containers of various types $t \in T$ to slots in container ship at each port $i = 1, \ldots, N$ in such a way that the stowage plan optimizes a weighted combination of objectives subject to constraints on a feasible stowage plan. We define set of containers C^t_{ij} that contains containers of type t that need to be transported from port i to port j. We assume that $T = \{$20ft, 40ft, 45ft$\}$ where each element represents a possible container size. Let (b, v, s) represent a slot in a container ship that is located in bay b, tier v and stack s. Then, we define $x^i_{c,bvs}$ to be 1 if the container ship departs port i with container c assigned to slot (b, v, s), and 0 otherwise. We denote the multi-port stowage plan by x. Finally, let w_c be the weight of each container c and N_i be the number of cranes available at each port i.

As discussed above, we focus on three components of the objective: $f_1(x)$, the cost of overstows generated by x; $f_2(x) = \mu_{trim} \max(f_{trim}(w_c, x) - f^*_{trim}, 0)$, the penalty costs associated with deviating from the optimal trim level, where if pre-determined level f^*_{trim} is exceeded, fuel consumption increases by μ_{trim} for each angle unit increase in trim; and $f_3(x) = \sum_{l=1}^{N_i} f_{crane(l)}(x)$, the total utilization of cranes, defined to be the proportion of time spent in operations by each crane relative to its idle time denoted by $f_{crane(l)}(x)$.

The constraints of the problem fall into several categories. Required containers must be transported, and assigned to a unique slot. Certain types of containers (such as refrigerated containers) must be assigned to specific types of spots. Slots must be filled from the bottom up – there can't be empty space below a container. Containers come in three principal sizes, 20, 40, and 45 feet, and there are storage constraints associated with the size of the container (for example, 40-ft containers cannot support two 20-ft containers). If a container is removed, all of the containers on top of it must first be removed. There are hatch lids on deck, and these hatch lids need to be removed to get at containers below these hatch lids, but containers on top of the hatch lids must first be removed in order to remove the hatch lids. Finally, vessel-related static constraints (attitude, stability, strength) must be satisfied. This typically means that associated measurements must fall between prescribed upper and lower bounds.

All of the constraints except the ones related to static measurements can be formulated as a linear-integer form at the expense of extra decision variables (see Gümüş and Kaminsky [2007] for details). However, the existence of integer variables and nonlinearities in the

Methodology	Papers
Simulation Based	Shields, 1984
Rule based	Davidor, 1998 Martin et.al., 1998 Wilson et.al, 1999 Wilson et.al, 2000 Dubrovsky et.al., 2002
Mathematical Model Based	Aslidis, 1998 Avriel et al., 1998

Table 1: Methodologies and models developed in the literature

problem makes it impractical to solve it directly using existing technology. Instead, we have developed a multi-stage decomposition approach for this problem.

Before discussing our solution approach, we briefly address the models and approaches developed both in the research literature and in industry. The practice in the literature has been to consider heuristic models. Wilson and Roach [2000] divide methodologies developed for the CSP into the following: simulation-based approaches combining simple loading heuristics with simulation methods, rule-based systems using the methods of artificial intelligence and metaheuristics, and mathematical-programming based approaches, using the methods of mathematical programming with relaxations of difficult constraints.

Table 1 summarizes the existing literature on the CSP. In addition, a recent review of operations research tools applied to problems faced in container terminal operations can be found in Steenken et al., 2004. In Gümüş and Kaminsky [2002], we proposed a 2-stage approach to this problem, which ultimately led to the four stage approach discussed in this paper.

3 The Decomposition Approach

We solve this problem using a four stage decomposition approach. In the first stage, the issue of crane efficiency is addressed while a worst case bound on the number of overstow is minimized. In the second stage, the number of overstows is further reduced, and finally in the third and fourth stages, the final stowage plan is generated.

The first two stages of the decomposition approach involve aggregating containers based on origin destination pairs. Define a transportation matrix (demand) D^t_{ij} that indicates how many containers of each type $t \in T$ need to be transported from port i to port j. The approach is detailed as follows:

Stage 1: In this stage, we assign a bay to each origin-destination type in order to minimize a worst case bound on the number of overstows. In particular, the output of this stage is an assignment of percentages of bays to particular aggregate origin destination pairs. For example, 62 percent of bay 3 might be assigned to containers going from port i to port j. In this stage, we also consider crane utilization at an aggregate level. Remember that there are N_i cranes available at each port. However, two adjacent cranes cannot work simultaneously on adjacent bays. This condition is satisfied if both origin and destination labels are assigned to form at least N_i equally sized clusters on the ship at each port i.

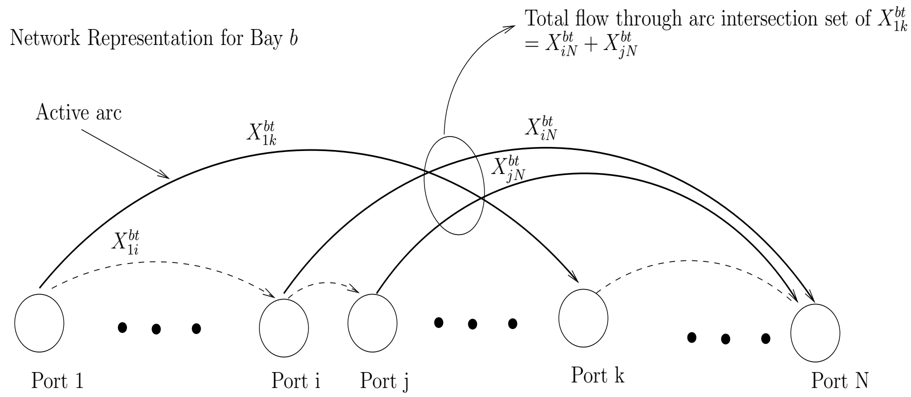

Figure 1: Representation of overstows with arc intersection sets. For a detailed explanation of the variables and concepts, see Gümüş and Kaminsky [2007]

To formulate this problem, we begin by defining independent topologically ordered networks for each bay $b, b = 1, 2, ..., B$ consisting of a node $i = 1, \ldots, N$ for each port 1 to N, where Port 1 corresponds the current port and port N corresponds the last port in the planning horizon. Next, for each pair of nodes (i, j), $i < j$, we define an arc from node i to j with an associated variable X_{ij}^{bt}. In our model, this variable will represent the percentage of bay b used for the purpose of stowing containers of type t originating from port i and heading to port j. The possible values for port index i is $1, 2, \ldots, N$. An example is shown in figure 1.

In Gümüş and Kaminsky [2007], we explain how we can use this network to characterize an upper bound on the number of overstows, we develop a related network that approximates the crane utilization for a particular stowage plan, and we explain how we can use these two networks to find efficient percentage allocations of container types as defined by origin/destination pairs to specific bays. To accomplish this task, we define a mixed-integer program which can be solved very quickly using existing technology. To give an example, for a ship with 8000 container capacity, it takes in the order of $1 - 2$ minutes to solve the first stage of the problem. A nice property of this decomposition is that this approach is *scalable* to instances of almost any size.

Stage 2: In this stage, we make the allocation decision for tiers, based on the output of the previous stage and accounting for hatch lids. Although space constraints prevent a complete discussion, some stacks are located both above and below deck, and the below-deck portion of sets of stack may be covered by a common hatch lid, so that the above-deck portions of all of these stacks need to be moved to access the below-deck portions of any of the stacks. This stage explicitly accounts for the impact of hatch lids on the number of restows.

Stage 3: In this stage, we make an assignment to slots, based on the output of Stage 2. (See Gümüş and Kaminsky [2007] for details.

Stage 4: Let S_{ij}^{t} be the set of slots that are assigned to a container of type $t \in T$ going from port i to port j. In this forth stage, we assign each $c \in C_{ij}^{t}$ to a slot $s \in S_{ij}^{t}$ in such a way that the hydro-static constraints f_h are satisfied. This is a difficult assignment problem with side constraints, so instead we solve the following problem for each origin-destination-type label i, j, t where $Z^c_{b,v,s}$ is a binary variable representing the assignment of a container to a slot:

$$P^A_{ijt} = \min_{Z^c_{b,v,s}: c \in C^t_{ij} \text{ and } (b,v,s) \in S^t_{ij}} \sum_{c \in C^t_{ij}} \sum_{(b,v,s) \in S^t_{ij}} \left[w_c(l_b - b)^2 + w_c(l_v - v)^2 + w_c(l_s - s)^2 \right] Z^c_{b,v,s} \tag{3.1a}$$

$$\text{s. t.} \quad \sum_{c \in C^t_{ij}} Z^c_{b,v,s} \le 1 \qquad \text{for all} (b, v, s) \in S^t_{ij} \tag{3.1b}$$

$$\sum_{(b,v,s) \in S^t_{ij}} Z^c_{b,v,s} = 1 \qquad \text{for all } c \in C^t_{ij} \tag{3.1c}$$

$$Z^c_{b,v,s} \in \{0, 1\} \tag{3.1d}$$

The constraints (3.1b)-(3.1c) define an assignment of each container c in C^t_{ij} to a slot (b, v, s) in S^t_{ij}. The mathematical program (3.1) has an objective that enforces a "smooth" weight distribution across the hull of ship around a preferred central point (l_b, l_v, l_s) in the container ship.

4 Conclusions

This novel approach to the CSP has several advantages over approaches developed in the past: this approach is *scalable* to instances of almost any size. Indeed, the first and second stages are designed specifically to achieve this goal. This is a *modular* approach. For each stage, the output of previous stage is used as an input of the next stage. This feature adds an extra dimension of freedom since the input and output of each stage can be monitored and even modified by the planner to deal with real-time changes in the data, and to account for additional constraints not included in the mathematical formulation. In addition, metaheuristics and improvement algorithms can be used on the output of each stage as time permits. Finally, this is a *hierarchical* approach that adds new elements to the stowage plan at each stage. This makes it easier to determine which of these elements make the problem particularly difficult for a specific instance.

In Gümüş and Kaminsky [2007], we present very successful preliminary computational results, and we are currently completing the analysis of results using additional industrial data. If this additional testing is as successful as our initial testing, this approach will be incorporated into commercial stowage planning software. We believe that automated stowage planning has the potential to significantly impact global maritime logistics.

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