Effective Distribution Policies Utilizing Logistics Contracting

Hyun-Soo Ahn • Osman Engin Alper • Philip Kaminsky

Stephen M. Ross School of Business, University of Michigan, 701 Tappan Street, Ann Arbor, Michigan 48109-1234, USA

Department of Industrial Engineering and Operations Research, University of California, Berkeley, California 94720-1777, USA

Department of Industrial Engineering and Operations Research, University of California, Berkeley, California 94720-1777, USA

hsahn@bus.umich.edu • engin@ieor.berkeley.edu • kaminsky@ieor.berkeley.edu

---

Logistics outsourcing is becoming a more widely utilized practice across many industries. Motivated by this observation, we develop models to analyze the optimal operation of a production-distribution system with stochastic demand and logistics outsourcing. For our initial investigation, we consider a just-in-time production-distribution system with linear production cost and a fixed commitment transportation contract. Under these assumptions, the optimal production policy is uniquely determined by the optimal shipping policy, so we are able to focus on the latter. We completely characterize the structure of the optimal policy, investigate the impact of contract and system parameters on overall system performance via a computational study, and summarize the practical implications of our results.

---

1. Introduction

Outsourcing non-core competencies has now become a widely accepted practice across many industries. Running an in-house trucking fleet is a difficult task and not a core competency of many firms. Accordingly, the number of firms that prefer to cooperate with outside companies for their logistics related responsibilities is growing rapidly (Alp et al. 2003). Outsourcing logistics operations is generally done via transportation contracts. Hence, acquiring transportation contracts and effectively utilizing them have become critical operational and tactical points of interest in supply chains, although there is limited research in this area. In this paper, we hope to contribute to this important area by starting to explore the characteristics of optimal distribution policies in the presence of logistics contracts.

Of course, production and distribution operations are generally the critical operational functions in manufacturing supply chains. Indeed, United States industry spends more than $350 billion on transportation and more than $250 billion on inventory holding costs annually (Lambert and Stock 1993). Although production and distribution operations can typically be decoupled if there is sufficient inventory between them, this approach leads to overall higher inventory levels and longer lead times in the supply chain than if these functions are more closely linked. Indeed, as supply chain inventory reduction efforts are becoming more and more common, the linkages between production and distribution are becoming tighter and tighter, requiring a significant focus on decision making in which production and distribution operations are optimized in an integrated manner. Our ultimate goal is to understand the operation of complex production and distribution systems; the work presented in this paper applies to just-in-time systems with simple linear production costs, and we are currently exploring generalizing this work to more complex production cost structures.

Contracts are an important area of study in law, economics and supply chain management. While the particular attention in law is on the legality and enforceability of various contractual structures, economics literature considers the incentives of the parties involved in the contract and the corresponding implications on the efficiency, social welfare and public policy. Bolton and Dewatripont (2005) and Tirole (1988) are excellent reviews on the substantial literature of contracts in economics. On the other hand, the supply chain management literature focuses on operational details such as the uncertainty in demand, material flow, expediting, penalties and inventory holding costs. Tsay et al. (1999) provides a detailed survey and a taxonomy of contracts in supply chain setting. In this literature, the main emphasis is on channel coordination and there are few studies that explicitly model transportation contracts. Alp et al. (2003) models a transportation contract design problem; Whittemore (1977) shows optimal ordering policies when there are two supply options, one being faster and more expensive than the other; Blumfeld et al. (1985) and Gallego and Simchi-Levi (1990) look at integrated inventory control and vehicle dispatching problems; Yano and Gerchak (1989) and Yano (1992) consider expedited shipments in their models, and determine optimal contract parameters assuming a base-stock ordering policy; Henig et al. (1997) deals with optimal ordering policies under a given supply contract.

In our model, which we detail in next section, we characterize the optimal policy of a production-distribution system under a given transportation contract when there is an expedited shipment service available. Our modeling approach is to some extent similar to Henig et al. (1997) in that we study the optimal behavior of the system under a given contract, but we model a just-in-time system and a general fixed commitment contract instead of a contract with constant reserved capacity in each period. This allows us to capture the delivery frequency of the transportation contract realistically as an integer multiple of the review period.

2. Model Development and Analysis

We consider a just-in-time manufacturing firm that must ship its product to some destination to meet stochastic demand, $w_t \geq 0$, through a finite planning horizon. For all the reasons mentioned in the previous section, this firm decides to outsource its logistics operations. At the beginning of the planning horizon, we assume that the firm is already in possession of a transportation contract. Transportation contracts often specify in advance the frequency and volume to be reserved by the logistics provider (Yano and Gerchak 1989). Thus, a fixed commitment contract in a finite horizon model can be specified by its shipment periods and its reserved capacities for these periods. We assume that the contract in hand has n shipments at periods $T_1, ..., T_n$ with reserved capacities $C_1, ..., C_n$ respectively. When the demand is uncertain, the transportation contract alone usually will not provide sufficient service levels. For this reason, in our model we utilize a spot market for shipping that provides expedited service (which is often the case in practice). At each period, an order can be shipped immediately through expedited shipment, which costs c per unit, or it can be delayed to utilize the contracted capacity in which case a waiting cost of p per unit applies every period. The discount factor is denoted by $\alpha \in (0, 1)$. We assume linear production costs with no lead time or capacities, which simplifies the production decision given the shipping decision, as all production will immediately precede shipment in quantities equal to the shipping quantities. Thus, to simplify subsequent notation, we do not explicitly model production in what follows. The decision problem is to find the optimal level of shipment u_t at each period given the number of pending orders x_t. The demand distributions for periods 0 through $T_n - 1$ are assumed to be mutually independent. At the end of the contract period, T_n, pending orders in excess of the contracted capacity, C_n, are shipped via expedited shipment service. We use the notation $(x)^+$ to represent $\max(0, x)$ throughout this section. The dynamic programming equations for this model follows.

$$J_{T_n}(x_{T_n}) = H_{T_n}(x_{T_n}) \tag{P}$$

$$J_t(x_t) = \min_{u_t \in [0, x_t]} \{H_t(u_t) + p(x_t - u_t) + \alpha \mathop{\mathrm{E}}_{w_t}[J_{t+1}(x_t - u_t + w_t)]\}, \qquad t = 0, ..., T_n - 1,$$

where $H_t(x) \equiv \begin{cases} c(x - C_t)^+, & \text{if } t \in \{T_1, ..., T_n\} \\ cx, & \text{otherwise.} \end{cases}$

In the next two subsections, we will analyze two special cases of this model, which will facilitate characterizing the optimal policy for the general case at the last subsection.

2.1 One shipment without capacity: $n = 1$, $T_1 = T$, $C_1 = C = \infty$.

In this case there is only one shipment period in the contract which has infinite capacity. The dynamic programming equations simplify as follows.

$$J_T(x_T) = 0 \tag{P1}$$
$$J_t(x_t) = \min_{u_t \in [0, x_t]} \{cu_t + p(x_t - u_t) + \alpha \operatorname*{E}_{w_t}[J_{t+1}(x_t - u_t + w_t)]\} \quad t = 0, ..., T-1.$$

The optimal policy is given by the following intuitive result, which says that it is optimal to ship every order immediately until we get sufficiently close to the end of the contract period, and after that it is optimal not to ship at all until the end of the contract period.

Proposition 1. *(Optimal Policy of (P1)) In* (P1)*, the optimal shipping quantity function, μ_t, is given by:*

$$\mu_t(x) = \begin{cases} x, & \text{if } p\frac{1-\alpha^{T-t}}{1-\alpha} > c; \\ 0, & \text{otherwise} \end{cases} \quad t = 0, ..., T-1.$$

2.2 One shipment with capacity: $n = 1$, $T_1 = T$, $C_1 = C < \infty$.

In this case we still have only one shipment period in the contract, but it now has a finite capacity, $C \geq 0$. The dynamic programming equations change as follows.

$$J_T(x_T) = c(x_T - C)^+ \tag{P2}$$
$$J_t(x_t) = \min_{u_t \in [0, x_t]} \{cu_t + p(x_t - u_t) + \alpha \operatorname*{E}_{w_t}[J_{t+1}(x_t - u_t + w_t)]\} \quad t = 0, ..., T-1$$

Notice that if $p \geq c$ then $\mu_t(x) = x$, or if $(1-\alpha)c \geq p$ then $\mu_t(x) = 0$, $t = 0, ..., T-1$. Thus, from now on we assume that $p < c$ and $(1-\alpha)c < p$ to avoid trivial cases. We also naturally assume that $x_0 \geq 0$.

The optimal policy is given by the following result.

Proposition 2. *(Optimal Policy of (P2)) In* (P2)*, for a given C, there is a sequence of increasing numbers $\{R_t\}_{t=0}^{T-1}$ between 0 and C such that $\mu_t(x) = (x - R_t)^+$ for $t = 0, ..., T-1$.*

These R_t values can be thought as "maximum levels of pending orders reserved by the future contracted shipments", and the policy can be interpreted as a "ship-down-to" type policy analogous to a base-stock policy in reverse. Here, the aim is to keep the level of pending orders at or below the reserved levels, which is similar to a base-stock policy keeping the inventory levels at or above the order-up-to levels.

2.3 Multiple shipments with capacity

The next theorem completely characterizes the optimal policy structure of the general problem (P) introduced earlier in this section.

Theorem 3. *(Optimal Policy of (P)) In* (P)*, for a given set of C_m and T_m, $m = 1, ..., n$, there is a sequence of sequence of increasing nonnegative numbers $\left\{\{R_t\}_{t=T_m}^{T_{m+1}-1}\right\}_{m=0}^{n-1}$ such that*

$$\mu_t(x) = \begin{cases} (x_t - R_t)^+, & \text{if } t \notin \{T_1, ..., T_n\} \\ \begin{cases} x, & \text{if } x \leq C_t \\ C_t + (x - C_t - R_t)^+, & \text{otherwise} \end{cases}, & \text{otherwise,} \end{cases}$$

where we define $T_0 \equiv 0$.

In words, we may say that the optimal policy is a "modified ship-down-to" type policy, where it is ship-down-to type in periods with no scheduled shipment, and in periods with scheduled shipment, the standing orders over the capacity of shipment is ship-down-to type.

The next result follows from Proposition 1 and provides a sufficient condition for the decomposition of the problem in time.

Proposition 4. *(Decomposition of (P)) In* (P)*, suppose that $p\frac{1-\alpha^{T_{N(t)}-t}}{1-\alpha} > c$ is satisfied for some t, where $N(t) = \min_{j\in\{1,...,n\}}(j|T_j > t)$. Then $R_t = 0$ and the problem decomposes at period $T_{P(t)}$, where $P(t) = \min_{j\in\{0,...,n\}}(j|T_j \leq t)$.*

3. Computational Study

We have completed an extensive computational study, and in this section we summarize our key observations, which provide some insight about the behavior of the optimal policy with respect to changing problem parameters. Firstly, we note that the ratio p/c and its size relative to the discount factor α are the key factors determining the ship-down-to levels, R_t. This is somewhat intuitive as the asymptotic behavior of the optimal policy as the reserved capacity goes to infinity is solely determined by these parameters. The smaller this ratio or the discount factor, the larger and flatter the $\{R_t\}$ sequence, which consequently renders a transportation contract more attractive for the manufacturer. Secondly, we observe that the mean of the demand distribution normalized with the reserved capacity has a scaling effect. In other words, as long as its ratio with the reserved capacity stays constant, the mean does not impact the shape of the $\{R_t\}$ sequence. If this ratio increases, the sequence becomes steeper, and if it decreases, the sequence becomes flatter. In general, the more flat the ship-down-to levels, the more profitable it is to use a contract. Thirdly, the reserved capacity has a vertical shifting effect on the ship-down-to-levels. An increase in the capacity evenly increases the positive portion of the $\{R_t\}$ sequence, and a decrease does the exact opposite. Lastly, we observed that the ship-down-to levels increase with increasing dispersion of the demand distribution when p/c ratio is small, making the sequence more flat; when p/c ratio is large, it makes the positive part of the $\{R_t\}$ sequence more concave, increasing the ship-down-to levels for the early periods and decreasing for the later.

4. Conclusion and Future Research Directions

In this study, we have completely characterized the optimal policy structure of a just-in-time production-distribution system under a fixed commitment transportation contract when production is uncapacitated, the production cost is linear, and the production lead time is zero. Our analysis and numerical study show that, for a manufacturing firm that wants to outsource its logistics operations, it is significantly more attractive to buy a transportation contract rather than using only the spot market when the ratio of pending order penalty to expedited shipping cost is small. However, when this ratio is large, especially in the case of large demand variance, a substantial increase in the frequency of shipments is necessary for the contact to be useful, which would probably render the contract price too expensive to be profitable.

In the future, we will extend this study to the case where there is positive lead time, economies of scale and capacity for production, which will make the production-distribution integration problem more complex and realistic. We also plan to analyze the impact of some other contract types such as option and flexibility contracts in this setting. It will also be interesting to include the contract procurement problem in the model. Given any price structure on the contract parameters, it is possible to incorporate the contract procurement problem in the manufacturer's model using the results obtained in this study. Finally, we also plan to consider the problem from the contract provider's point of view and model its resource allocation, contract design and pricing problems in a competitive market.

Overall, the models we analyzed in this study provide some insight to the practical use of transportation contracts and, more importantly, provide a foundation for future investigation of models that incorporate more complicated critical aspects of important real-world problems relating to integrated production and distribution management in the presence of logistics contracts.

References

Alp, O., N.K. Erkip, R. Gullu. 2003. Outsourcing logistics: designing transportation contracts between a manufacturer and a transporter. *Transportation Science* **37** 23–39.

Bolton, P., M. Dewatripont. 2005. *Contract Theory.* The MIT Press, Cambridge, CA.

Burns, L.D., R.W. Hall, D.E. Blumenfeld, C.F. Daganzo. 1985. Distribution strategies that minimize transportation and inventory costs. *Operations Research* **33** 469–490.

Gallego, G., D. Simchi-Levi. 1990. On the effectiveness of direct shipping strategy for the one-warehouse multi-retailer R-systems. *Management Science* **36** 240–243.

Henig, M., Y. Gerchak, R. Ernst, D.F. Pyke. 1997. An inventory model embedded in designing a supply contract. *Management Science* **43** 184–189.

Lambert, D.M., J.R. Stock. 1993. *Strategic Logistics Management.* McGraw Hill, New York, NY.

Tirole, J. 1988. *The Theory of Industrial Organization.* The MIT Press, Cambridge, CA.

Tsay, A.A., S. Nahmias, N. Agrawal. 1999. Modeling supply chain contracts: a review. S. Tayur, R. Ganeshan, M. Magazine, eds. *Quantitative Models for Supply Chain Management.* Kluwer's International Series, Boston, MA. 299–336.

Whittemore, A.S., S.C. Saunders. 1977. Optimal inventory under stochastic demand with two supply options. *SIAM Journal on Applied Mathematics* **32** 293–305.

Yano, C.A. 1992. Optimizing transportation contracts to support just-in-time deliveries: the case of one contracted truck per shipment. *IIE Transactions* **24** 177–183.

Yano, C.A., and Y. Gerchak. 1989. Transportation contracts and safety stocks for just-in-time deliveries. *Journal of Manufacturing and Operations Management* **2** 314–330.