Centralized Versus Decentralized Scheduling and Due Date Quotation in a Make-to-Order Supply Chain

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1. Introduction

Although there is a vast and growing amount of literature on supply chain coordination, the majority of this literature focuses on make-to-stock systems and performance measures built around service and inventory levels. On the other hand, an increasing number of supply chains are better characterized as make-to-order systems. Fast and on-time delivery is obviously critical for any firm that operates as a make-to-order system, and the manufacturer-supplier relationship dramatically impacts the delivery of products and system performance. In this research, we analyze such a make-to-order system in a supply chain setting and develop algorithms for scheduling and due-date quotation in both centralized and decentralized versions of this system. We also explore the value of information sharing in this setting.

We consider a manufacturer with a single supplier that has to quote due dates to arriving customers in a make-to-order production environment. The manufacturer is penalized for long lead times, and for missing due dates. In order to meet due dates, the manufacturer has to obtain components from the supplier. As in most single facility due date quotation models (see Kaminsky and Hochbaum [1] for a survey), the manufacturer's objective is to determine a schedule and quote due dates in order to minimize a function of quoted lead time length and lateness. We consider several variations of this problem, and design effective due-date quotation and scheduling rules for centralized and decentralized versions of the model. We also investigate the relative advantage of a centralized system under various conditions.

In our model, customer orders i, $i = 1, 2, ..., n$ arrive at the manufacturer at time r_i which is not known in advance, and the manufacturer quotes each customer a due date d_i at the arrival of the order. To complete the processing and to deliver the product to the customer, the manufacturer needs some materials from the supplier and orders these materials from the supplier at the time of the order arrival. We assume that there are k types of jobs with different processing times p_i^1 at the supplier and p_i^2 at the manufacturer, for $i = 1, 2...k$.

Our objective is to determine an effective online scheduling and due date quotation algorithm for the manufacturer and the supplier for any n job instance to minimize the total cost function $Z_n = \sum_{i=1}^{n} (c^d * d_i + c^t * T_i)$ where $T_i = (C_i - d_i)^+$ is the tardiness of job i and c^d and c^t are the unit due date and tardiness costs where $c^t > c^d$. Although sequencing and due date quotation are clearly interrelated decisions, our strategy in developing algorithms is to first try to determine an optimal schedule to minimize the total completion times of the jobs, and then to find a due-date quotation algorithm that effectively matches these completion times. However, we know that the simplest version of this problem is similar to the problem of minimizing total completion times for a single machine with release times, $1/r_i/\sum C_i$, which is NP-Hard (see Pinedo [2]). Thus, we are motivated to use a probabilistic approach, and to develop asymptotically optimal solution to this problem; that is we look for relative optimality as the number of jobs goes to infinity, because we have found in previous research that asymptotically optimal algorithms frequently work well for smaller instances of the same problem.

We develop three different models for this problem, and we use these three models to begin to quantify the value of information and centralization; specifically, we find the difference between objective values for centralized and decentralized systems. In our first model, the centralized model, the manufacturer and the supplier work together and a single decision agent makes all the decisions, utilizing all the system information to minimize total supply chain costs. In the second model, the simple decentralized model, the manufacturer works independently from the supplier and both parties work to minimize their own costs. The manufacturer has no information about the supplier but still needs to make the scheduling decisions and quote a due date to the customers at the time of the order arrivals. In the third model, the decentralized model with information exchange, the manufacturer still works independently from the supplier but the supplier provides some information about his own processes by quoting a due date to the manufacturer. The manufacturer can thus quote better due dates to his customers using this information. In all these models, we assume that all customer orders must be accepted, and that all orders will be placed regardless of the quoted due-date.

2. Single Facility Model

As a building block for later results, we first analyze a single facility system. For this single facility model, it is known that processing the shortest available job when the facility is idle, the SPTA scheduling rule, asymptotically minimizes the total completion times of the jobs, so we use this rule and develop the following due date quotation heuristic:

$$d_i = r_i + minwait_i + slack_i$$

where *minwait* value is defined as the remaining time of the job in process plus the total processing times of all the jobs in front of job i and $slack_i$ is some additional time added to the due date in order to account for future arrivals with processing times less than this job. This slack value is calculated as below:

If $pr_i * EP_i \geq ET$, $slack_i = (n - i) * pr_i * EP_i$
else $slack_i = min\{\frac{minwait_i pr_i EP_i}{ET - pr_i EP_i}, (n - i) pr_i EP_i\}$

where pr_i is the probability that an arriving job has a processing time less than p_i, EP_i is the expected processing time of a job given that it has processing time less than p_i and ET is the expected interarrival time. We show that this scheduling and due date quotation heuristic, called SPTA-SL, is asymptotically optimal and prove the following theorem.

Theorem 1 *Consider a series of randomly generated problem instances meeting the requirements described above. Almost surely,*

$$\lim_{n \to \infty} \frac{Z_n^{SPTA-SL} - Z_n^*}{Z_n^*} = 0$$

3. Centralized Supply Chain Models

We next extend the single facility results to a supply chain setting. Scheduling the jobs according to SPTA in the first facility and FCFS at the second facility is asymptotically optimal for minimizing the total completion time under the conditions of our model [3]. Based on this observation, we develop the following asymptotically optimal due date quotation algorithm for this centralized model:

$$d_i^2 = d_i^1 + p_i^2 + max\{t_i^{21} + t_i^{22} + slack_i^2 - (d_i^1 - r_i), 0\}$$

Above, $t_i^{21} = \sum_{i \in A} p_i^2$ and $t_i^{22} = \sum_{i \in B} p_i^2$ where A is the set of jobs in the facility 1 queue scheduled before job i and B is the set of jobs in the facility 2 queue at time r_i. Also, d_i^1 is calculated as in the single facility model and $slack_i^2$ is calculated as follows:

$$slack_i^2 = min\{\frac{(d_i^1 - r_i - p_i^1)}{ET} pr\{p < p_i\}, (n-i)pr\{p < p_i\}\}E\{p^2|p < p_i\}$$

We prove the following result for this scheduling and due-date quotation heuristic, called $SPTA_p - SLC_1$:

Theorem 2 *For a series of randomly generated problem instances of size n, if processing times are independent and exchangeable, then almost surely,*

$$\lim_{n \to \infty} \frac{Z_n^{SPTA_p - SLC_1} - Z_n^*}{Z_n^*} = 0$$

If the processing times are not exchangeable, we modify the sequencing and due date quotation rules slightly, and show that these modifications are asymptotically optimal. If $EP_1 \geq EP_2$, for example, we utilize the following algorithm for sequencing:

1. Find the critical processing time p_c^1 such that $E[p_2] = E[p_1 | p_1 \leq p_1^c]$
2. Let G_1 be the set of jobs with $p_1^i \leq p_1^c$ and let G_2 be the set of jobs with $p_1^i > p_1^c$. Whenever the first facility is empty, process the job with minimum total processing time from set G_1 at the first facility. If G_1 is empty, process the job with minimum p_1 from G_2 on the first machine.
3. Process the jobs at the second facility according to FCFS rule.

Observe that the jobs in G_2 satisfy the property $min\{p_1 : p_1 \epsilon G_2\} > EP_2$. We prove:

Theorem 3 *For any n job instance, if $min\{p_1\} > EP_2$, then the algorithm described above is asymptotically optimal for minimizing total completion time.*

4. Decentralized Supply Chain Models

We also consider both a simple decentralized model, and a decentralized model with some information exchange. Since the manufacturer and the supplier work independently from each other in the decentralized models, the SPTA sequencing rule, which is asymptotically optimal to minimize completion times in single facility systems, is used for both parties.

In the simple decentralized model the manufacturer has no information about the status of the supplier, so he has to estimate the delay at the supplier by using the only available information, the number of jobs at the supplier side, denoted q_i^1. Note that this is equal to the total number of jobs that have arrived minus the number of jobs delivered to him by the supplier at the time of the arrival of job i. Thus, we use the following due-date quotation rule for the manufacturer:

$$d_i = d_i^1 + p_i^2 + eq_i^2 + slack_i^2$$

To use this rule, the manufacturer needs to approximate the value of d_i^1 using q_i^1. Above, eq_i^2 denotes the approximate queue length in front of job i when it arrives at the manufacturer from the supplier and $slack_i^2$ is the approximate length of the jobs that will arrive at the manufacturer after job i but will be processed there before job i. We calculate these values as in the centralized model, and prove the following theorem for this scheduling and due-date quotation heuristic, which we call $SPTA - SLC_{SD}$.

Theorem 4 *$SPTA-SLC_{SD}$ heuristic is asymptotically optimal given manufacturer's knowledge in the decentralized case, that is, almost surely,*

$$\lim_{n \to \infty} \frac{Z_n^{SPTA-SLC_{SD}} - Z_n^*}{Z_n^*} = 0$$

We also consider a decentralized model with information exchange, in which, the d_i^1 value in the above equation is given by the supplier, who clearly has more information about his processes, and so can better estimate this value. Thus, the manufacturer can quote a more accurate due date to his customers. We develop a heuristic, $SPTA - SLC_{DIE}$, for this case, and prove:

Theorem 5 *For a series of randomly generated problem instances of size n, given manufacturer's knowledge, the $SPTA - SLC_{DIE}$ heuristic is asymptotically optimal for the decentralized case with information exchange, that is almost surely,*

$$\lim_{n \to \infty} \frac{Z_n^{SPTA-SLC_{DIE}} - Z_n^*}{Z_n^*} = 0$$

5. Computational Experiments

For the centralized model, we designed several computational experiments with different mean processing times for both parties to investigate the effectiveness of these asymptotically optimal heuristics even for relatively small problem instances. We observe that as the number of jobs increases, the ratio of the total due date plus lateness over the total actual completion times goes to 1 and the total tardiness over the total actual completion time ratio is seen to be converging to 0 as it should be. Also, even for smaller instances with 10 or 100 jobs, these ratios are very close to 1 and 0, respectively, suggesting that these asymptotically optimal algorithms also work well for smaller number of jobs.

We also conducted a computational study to investigate the performance differences between the centralized and decentralized systems. We observe that when the mean processing time is small at the supplier, i.e. when there is no or very little congestion at the supplier side, the centralized and decentralized models give very similar results. However, as the congestion at the supplier starts to increase, the value of centralization also increases, and the centralized model begins to significantly outperform the decentralized models. Also, as the mean processing time at the manufacturer increases relative to the mean processing time at the supplier, the value of centralization decreases. Apparently, as the congestion in the supplier becomes higher relative to the congestion at the manufacturer, the lack of control of the supplier performance and detailed information about supplier status has a greater impact on the manufacturer in the decentralized system. However, if processing at the manufacturer dominates system performances, additional information about supplier performance or control over the supplier has has little impact.

Finally, we compare both decentralized models under a variety of conditions. In general, we observe that under some conditions the decentralized model with information exchange gives results that are as much as 30% better than the simple decentralized model. In cases where centralization is impractical or impossible, a relatively simple information exchange can have a dramatic impact on system performance.

References

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[3] Xia, C., G. Shanthikumar, and P. Glynn (2000), On The Asymptotic Optimality of The SPT Rule for The Flow Shop Average Completion Time Problem. *Operations Research* **48**, pp. 615-622.