A Single Phase Dynamic Program with Independent Production Decision for Production-Capacitated Two- and Multi-Stage Lot-Sizing Problems

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1 Introduction

We consider a multi-stage capacitated lot-sizing problem (MLSP-PC), where the goal is to generate a centralized production and distribution plan for a supply chain which consists of a manufacturer with finite production capacity, intermediate agents and a retailer facing deterministic demand at the last stage. Clearly, centralized planning of an entire supply chain will lead to a lower-cost production and distribution plan than decentralized planning by independent agents. However, one of the obstacles to the centralized planning is the time to optimality due to the increased problem complexity from the simultaneous consideration of all problem parameters. Thus, a key to reducing this complexity is to allow decentralized or independent decisions of each agent whenever doing so will not negatively impact the overall solution. One of the purposes of this paper is to show that production decisions at the first stage of the supply chain can be made independently from transportation decisions at other stages in certain settings without negatively impacting the solution. This independence of the first-stage decision from other stages leads us to first focus on the two-stage problem (2LSP-PC) in which only two agents exist, the manufacturer and the retailer. The multi-stage problem with certain cost structures can be addressed in a similar fashion.

The single-stage uncapacitated lot-sizing problem for a manufacturer was introduced by Wagner and Whitin (1958), and efficient solution algorithms were designed by Federgruen and Tzur (1991), Wagelmans et al. (1992) and Aggarwal and Park (1993). The multi-stage version of the uncapacitated problem was solved by Zangwill (1969). To deal with the manufacturer's production capacity, Florian and Klein (1971) solved the capacitated single-stage lot-sizing problem (See also van Hoesel and Wagelmans 1996). Optimal algorithms for the multi-stage problem accounting for production capacity are provided by van Hoesel et al. (2005) and Hwang et al. (2011).

The MLSP-PC in general assumes concave production, transportation and inventory carrying costs through the planning horizon and the supply chain. As a special case of concave cost structure, the so-called *non-speculative* (transportation) cost structure assumes that inventory holding cost functions are linear and each of transportation cost functions consists of a fixed setup cost and per-unit transportation cost in which no speculative motive is allowed for keeping inventory. If each transportation does not incur setup cost then the supply chain is said to be have *linear* transportation costs. For the 2LSP-PC with the length of planning horizon T, Kaminsky and Simchi-Levi (2003) developed an $O(T^8)$ algorithm for their class of concave transportation costs and van Hoesel et al. (2005) presented three algorithms with complexities $O(T^7)$, $O(T^6)$ and $O(T^5)$ for concave, non-speculative transportation and linear transportation cost structures, respectively. Van Hoesel et al. (2005) also put the complexity reduction of their algorithms as an open question. In this paper, we address this question by deriving $O(T^6)$, $O(T^5)$ and $O(T^4)$ algorithms for concave, non-speculative transportation and linear transportation cost structures in the 2LSP-PC, respectively. For the multi-stage production capacitated problem with non-speculative costs, we present an efficient $O(T^6)$ algorithm as compared to the $O(T^7)$ algorithm of van Hoesel et al. (2005).

We note that all the algorithms in this paper are efficient by a factor of $O(T)$ over the best known ones until now. Most of these improvements are made by making possible separating the production decision from transportation decisions. To support such independence, it is crucial to have an appropriate dynamic programming algorithm. We adapt the single-phase dynamic programming approach first developed in Hwang et al. (2011) for multi-stage problems.

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