A Polynomial Algorithm for the Multi-Stage Production-Capacitated Lot-Sizing Problem

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The multi-stage lot-sizing problem with production capacities (MLSP-PC) deals with a supply chain that consists of a manufacturer with stationary production capacity and intermediaries (distribution centers or wholesalers) and a retailer to face deterministic demand. An optimal supply chain plan for the MLSP-PC specifies when and how many units each organization of the supply chain has to produce or transport to ultimately fulfill the demand at the retailer with the objective of minimizing total supply chain cost. All the production, transportation and inventory holding costs in each organization are assumed to be concave.

The single-stage uncapacitated lot-sizing problem for a manufacturer was introduced by Wagner and Whitin [6] and the multi-stage version of the uncapacitated problem was solved by Zangwill [5]. To address the manufacturer's production capacitated situation, Florian and Klein [1] solved the single-stage capacitated lot-sizing problem. Optimal algorithms for the multi-stage problem with production capacity were first presented by Kaminsky and Simchi-Levi for the two-stage case (2LSP-PC) [2]. Van Hoesel et al. [3] generalized the 2LSP-PC to the multi-stage lot-sizing problem MLSP-PC.

For the multi-stage dynamic lot-sizing problem with production capacities, Van Hoesel et al. [3] provide an $O(LT^4 + T^7)$ algorithm when no speculative motive exits in transportation (we call it the MLSP-PC with *non-speculative cost structure*) where L is the number of stages in the supply chain and T is the length of the planning horizon. For the most general MLSP-PC problem with concave cost structure, however, no polynomial time algorithm has been presented until now. Although the non-speculative cost structure explains quite well the value-adding phenomena from upstream to downstream operations in a supply chain, it has limitation in modeling the economies of scale in general, for instance, the quantity discount for large shipment in transportation industry. To address the economies of scale in a more realistic supply chain, we need to attack the MLSP-PC with concave costs in all the stages. The primary purpose of this paper is to provide a polynomial time algorithm for the MLSP-PC with concave costs both in the number of stages L and the length of horizon T.

Most dynamic lot-sizing problems are modeled as discrete-time dynamic programming, and are solved by iteratively enumerating over time periods. For instance, when solving the single-stage capacitated problem defined in Florian and Klein (1971), one needs to solve the optimality equation for each state (that is, cumulative production quantity) in a given period. Then, the same computations are repeated for each subsequent period to determine the optimal policy and the resultant production schedule. This is reflected by the fact that we often use time as a subscript of the value function. Indeed, van Hoesel et al. (2005) show that a traditional time-based enumeration solves the MLSP-PC with non-speculative transportation costs in polynomial time, using the fact that this multi-stage lot sizing problem with fixed-charge transportation and linear inventory costs is fully specified by characterizing manufacturing decisions. However, under a general concave cost structure, manufacturing decisions no longer characterize the entire plan. In order to solve the DP, we need to keep track of production and transportation decisions at all stages. Consequently, there is no polynomial algorithm that will solve this problem by performing recursive calculations over the time periods only.

On the other hand, in this paper, we propose a different way to conduct iterative computations to solve the MLSP-PC. Instead of iterating over time, we iterate along path in the two dimensional space of time and stage in the supply chain, which we call a *basis path*. Consequently, in contrast to every other lot-sizing DP that we are aware of, our algorithm requires us to in general iterate both forward and backward in time. We first present a DP algorithm for the case where a basis path given. The path contains *basis nodes* of pairs of stage and period over which the DP iterates. By exploiting the structures derived from consecutive basis nodes, we establish optimality equations with immediate costs to evaluate the value function for each basis node. This algorithm does not directly yield a polynomial algorithm since there are a large (indeed, exponential) number of basis paths. However, because the evaluation of the immediate cost at each basis node depends on its neighbor basis nodes not on the entire basis path, this allows for focusing on a sufficiently small set of possible basis paths, leading to a polynomial time algorithm that solves the MLSP-PC with general concave costs in $O(LT^{10})$ time. In addition to this effective solution methodology, we improve the algorithm to run in $O(LT^8)$ time by efficiently evaluating costs associated with basis paths.

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